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Extinction for a Fast Diffusion Equation Involving a Nonlinear Gradient Source and a Singular Potential

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Abstract: This article deals with the extinction behavior of the solution for a fast diffusion equation involving a nonlinear gradient source and a singular potential. Based on the energy estimate method, the Hardy-Littlewood-Sobolev inequality, and the ordinary differential inequality approach, we obtain the sufficient conditions on the occurrence of the finite time extinction phenomenon of the weak solution. Meanwhile, we establish the existence of a non-extinction weak solution via constructing suitable non-extinction sub- and super-solutions and applying the standard iteration process.

Key words: extinction; fast diffusion equation; gradient source; singular potential

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0 Introduction

The aim of the present article is to deal with the critical extinction exponent of the solution to the following fast diffusion equation involving both a nonlinear gradient source and a singular potential

$$\begin{cases} |x|^{-s}u_t=\operatorname{div}\left(u^{\alpha}|\nabla u|^{m-1}\nabla u\right)+\lambda|\nabla u|^{q}, & (x,t)\in\Omega\times(0,+\infty),\\ u(x,t)=0, & (x,t)\in\partial\Omega\times(0,+\infty),\\ u(x,0)=u_0(x), & x\in\bar{\Omega}, \end{cases} \tag{1}$$

where $s>0$, $m\geqslant 0$, $\lambda>0$, $q>0$, $m+\alpha\in(0,1)$, $\Omega\subset\mathbb{R}^N$ is a bounded domain with smooth boundary $\partial\Omega$, $N>\max\{s,m+1\}$, $|x|=\sqrt{x_1{}^2+\cdots+x_N{}^2}$ for $x=(x_1,\cdots,x_N)\in\Omega$, $u_0\in L^{\infty}(\Omega)\cap W_0{}^{1,m+1}(\Omega)$ is a non-negative and non-trivial function. It is well-known that model (1) with $s=\alpha=0$, $m=1$ and $q=2$ becomes the classical Kardar-Parisi-Zhang equation, originated from the physical theory of growth and roughening of surfaces[1].

In Refs. [2-3], the author proved the local existence result of the weak solution for problem (1). It is worth mentioning that the comparison principle for problem (1) can be established by Theorem 3.9 in Ref. [4] (or Subsection 1.1

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in Ref. [5]). Finite time extinction phenomenon, as one of the most remarkable properties that distinguish nonlinear parabolic problems from the linear ones, has received considerable attention and interest from mathematicians in the past two decades[6-10]. In particular, the extinction and non-extinction behaviors of the following fast diffusion model have been studied by many scientists,

$$\begin{cases} |x|^{-s}u_t=\operatorname{div}\left(u^{\alpha}|\nabla u|^{m-1}\nabla u\right)+f(x,t,u,\nabla u), & (x,t)\in\Omega\times(0,+\infty),\\ u(x,t)=0, & (x,t)\in\partial\Omega\times(0,+\infty),\\ u(x,0)=u_0(x), & x\in\bar{\Omega}, \end{cases} \tag{2}$$

where $s\geqslant 0$, $m>0$ and $m+\alpha\in(0,1]$. Gu[11] considered model (2) with $s=\alpha=0$, $m=1$ and $f(x,t,u,\nabla u)=-u^p$. He proved that the non-trivial solution of problem (2) vanishes in finite time if and only if $p\in(0,1)$, and explained how the strong absorption term causes the finite time extinction phenomenon from the physical point of view. When $s=0$, $m+\alpha\in(0,1)$ and $f(x,t,u,\nabla u)=\lambda u^p$ with $\lambda>0$, Tian *et al*[12-14] concluded that the critical extinction exponent of model (2) is $p=m+\alpha$. When $s=0$, $m+\alpha\in(0,1)$ and $f(x,t,u,\nabla u)=\lambda|\nabla u|^p$ with $\lambda>0$, Mu *et al*[15-16] concluded that if $m+\alpha<p<\dfrac{m+1}{2-\alpha}$, then the nonnegative weak solution of model (2) will vanish in finite time for appropriately small initial data; if $p<m+\alpha$, then the nonnegative weak solution of model (2) does not possess finite time extinction phenomenon for appropriately large λ. Very recently, Liu and Yang[17] investigated the extinction and non-extinction properties of the solution to model (2) with $\alpha=0$, $s\in(0,m+1)$ and $f(x,t,u,\nabla u)=\lambda|\nabla u|^p$ with $\lambda>0$.

As far as we know, there is no result on extinction behavior for problem (1) with $s>0$. Our interest is to deal with the effects of singular potential $|x|^{-s}$, the competitive interaction among the fast diffusion term and the gradient source term on the extinction behavior for problem (1).

1 Preliminaries and Main Results

It is noted that problem (1) might be degenerate or singular at the points where $u=0$ or $\nabla u=0$. Throughout this article, we work mostly with the weak solution to problem (1) in the following sense.

Definition 1 For some $T>0$, a function $u(x,t)$ defined in $\overline{\Omega}\times[0,T)$ is called a weak sub-solution (resp. super-solution) of problem (1) if it fulfills the following conditions:

(i) $u^{\alpha}|\nabla u|^{m+1}\in L^1(0,T;L^1(\Omega))$, $u\in C(0,T;L^{\infty}(\Omega))\cap L^q(0,T;W^{1,q}(\Omega))$ and $|x|^{-s}u_t\in L^2(0,T;L^2(\Omega))$;

(ii) for any non-negative test function $\xi\in C_0^{\infty}(\bar{\Omega}\times[0,T])$, one has

$$\int_0^T\int_{\Omega}\left(|x|^{-s}u_t\xi+u^{\alpha}|\nabla u|^{m-1}\nabla u\cdot\nabla\xi\right)\mathrm{d}x\mathrm{d}t\leqslant(\text{resp.}\geqslant)\int_0^T\int_{\Omega}\lambda|\nabla u|^q\xi\,\mathrm{d}x\mathrm{d}t\ ; \tag{3}$$

(iii) $u(x,t)\leqslant(\text{resp.}\geqslant)\ 0$ in $\partial\Omega\times(0,T)$;

(iv) $u(x,0)\leqslant(\text{resp.}\geqslant)\ u_0$ in $\bar{\Omega}$.

A function $u(x,t)$ is a weak solution of problem (1) if it is both a sub-solution and a super-solution of problem (1).

Our main results are the following three theorems.

Theorem 1 Assume that $0<m+\alpha<q<\dfrac{m+1}{2-\alpha}$, $0<s<m+1$ and $u_0(x)$ is appropriately small. Then the nonnegative weak solution of problem (1) vanishes in finite time.

Theorem 2 Assume that $0<q<m+\alpha<1$. Then for some appropriate Ω, problem (1) at least possesses a non-extinction weak solution.

Theorem 3 Assume that $0<m+\alpha=q<1$. Then the extinction phenomenon will occur for $m=0$ and appropriately small λ, while problem (1) at least exists a non-extinction weak solution for appropriately large λ.

Remark 1 From Theorems 1, 2 and 3, it follows that $q_c=m+\alpha$ is the critical extinction exponent of the weak solution to problem (1).

2 Proofs of the Main Results

In this section, the sufficient conditions on the occurrence of the finite time extinction phenomenon of the weak solution, and the existence of the non-extinction weak solution will be discussed.

Proof of Theorem 1 Take

$$l>\max\left\{\frac{N(1-m-\alpha)+s(m+\alpha)-(m+1)}{m+1-s},0\right\}. \tag{4}$$

Multiplying both sides of the first equation in (1) by u^l, and integrating over Ω by parts, one has

$$\frac{1}{l+1}\frac{\mathrm{d}}{\mathrm{d}t}\int_\Omega |x|^{-s}u^{l+1}\mathrm{d}x+l\left(\frac{m+1}{m+\alpha+l}\right)^{m+1}\int_\Omega\left|\nabla u^{\frac{m+\alpha+l}{m+1}}\right|^{m+1}\mathrm{d}x=\lambda\int_\Omega u^l|\nabla u|^q\mathrm{d}x. \tag{5}$$

Since $q<m+1$, Young's inequality with ε can be used to obtain

$$\begin{aligned}\int_\Omega u^l|\nabla u|^q\mathrm{d}x&=\left(\frac{m+1}{m+\alpha+l}\right)^q\int_\Omega u^{\frac{l(m+1-q)-q(\alpha-1)}{m+1}}\left|\nabla u^{\frac{m+\alpha+l}{m+1}}\right|^q\mathrm{d}x\\&\leqslant\varepsilon\left(\frac{m+1}{m+\alpha+l}\right)^q\int_\Omega\left|\nabla u^{\frac{m+\alpha+l}{m+1}}\right|^{m+1}\mathrm{d}x+C(\varepsilon)\left(\frac{m+1}{m+\alpha+l}\right)^q\int_\Omega u^{\frac{l(m+1-q)-q(\alpha-1)}{m+1-q}}\mathrm{d}x.\end{aligned} \tag{6}$$

Taking $\varepsilon\in\left(0,\ \frac{l}{\lambda}\left(\frac{m+1}{m+\alpha+l}\right)^{m+1-q}\right)$ such that

$$C_1=(l+1)\left(\frac{m+1}{m+\alpha+l}\right)^q\left[l\left(\frac{m+1}{m+\alpha+l}\right)^{m+1-q}-\lambda\varepsilon\right]>0,$$

and substituting (6) into (5), one can claim that

$$\frac{\mathrm{d}}{\mathrm{d}t}\int_\Omega |x|^{-s}u^{l+1}\mathrm{d}x+C_1\int_\Omega\left|\nabla u^{\frac{m+\alpha+l}{m+1}}\right|^{m+1}\mathrm{d}x\leqslant C_2\int_\Omega u^{\frac{l(m+1-q)-q(\alpha-1)}{m+1-q}}\mathrm{d}x, \tag{7}$$

where $C_2=\lambda C(\varepsilon)(l+1)\left(\frac{m+1}{m+\alpha+l}\right)^q$.

Since Ω is a bounded domain in $\mathbb{R}^N$, one knows that there exists a ball $B(0,R)\subset\mathbb{R}^N$ centered at $x=0$ with radius R such that $\Omega\subseteq B(0,R)$. Then, by a series of calculations, one can deduce that

$$\int_\Omega |x|^{-s}\mathrm{d}x\leqslant\int_{B(0,R)}|x|^{-s}\mathrm{d}x=\frac{N\pi^{\frac{N}{2}}}{\Gamma\left(\frac{N}{2}+1\right)}\int_0^R r^{N-s-1}\mathrm{d}r=\frac{N\pi^{\frac{N}{2}}R^{N-s}}{(N-s)\,\Gamma\left(\frac{N}{2}+1\right)}=:C_3, \tag{8}$$

where $\Gamma(\cdot)$ is the usual Gamma function.

For convenience, we set $a=\frac{N-(m+1)}{\theta(N-s)}$ with $\theta=\frac{l+m+\alpha}{l+1}\in(0,1)$. By (4) and $N>s$, one can verify that $a\in(0,1)$. Furthermore, making use of Hölder's inequality, (8), and Hardy-Littlewood-Sobolev inequality[18-19], one has

$$\begin{aligned}\int_\Omega |x|^{-s}u^{l+1}\mathrm{d}x&\leqslant\left(\int_\Omega\left[|x|^{-sa}\left(u^{\frac{m+\alpha+l}{m+1}}\right)^{\frac{m+1}{\theta}}\right]^{\frac{1}{a}}\mathrm{d}x\right)^a\left(\int_\Omega\left[|x|^{-s(1-a)}\right]^{\frac{1}{1-a}}\mathrm{d}x\right)^{1-a}\\&\leqslant C_3^{1-a}\left(\int_\Omega|x|^{-s}\left(u^{\frac{m+\alpha+l}{m+1}}\right)^{\frac{(m+1)(N-s)}{N-(m+1)}}\mathrm{d}x\right)^a\leqslant C_3^{1-a}C_4^a\left(\int_\Omega\left|\nabla u^{\frac{m+\alpha+l}{m+1}}\right|^{m+1}\mathrm{d}x\right)^{\frac{1}{\theta}},\end{aligned}$$

which implies that

$$\int_\Omega\left|\nabla u^{\frac{m+\alpha+l}{m+1}}\right|^{m+1}\mathrm{d}x\geqslant C_3^{\theta(a-1)}C_4^{-a\theta}\left(\int_\Omega|x|^{-s}u^{l+1}\mathrm{d}x\right)^\theta, \tag{9}$$

where C_4 is the constant in Hardy-Littlewood-Sobolev inequality. On the other hand, put

$$\eta=\frac{l(m+1-q)-q(\alpha-1)}{(m+1-q)(l+1)}\in(0,1).$$

It follows from $q>m+\alpha$ that $\theta<\eta$. Using Hölder's inequality again, one has

$$\int_\Omega u^{\frac{l(m+1-q)-q(\alpha-1)}{m+1-q}}\mathrm{d}x\leqslant|\Omega|^{1-\eta}\left(\int_\Omega u^{l+1}\mathrm{d}x\right)^{\eta}\leqslant|\Omega|^{1-\eta}R^{s\eta}\left(\int_\Omega|x|^{-s}u^{l+1}\mathrm{d}x\right)^{\eta}. \tag{10}$$

Letting $g(t)=\int_\Omega|x|^{-s}u^{l+1}\mathrm{d}x$ and substituting (9), (10) into (7), one has

$$\frac{\mathrm{d}}{\mathrm{d}t}g(t)+C_5g^{\theta}(t)\leqslant C_6g^{\eta}(t), \tag{11}$$

which is equivalent to

$$\frac{\mathrm{d}}{\mathrm{d}t}g(t)+\frac{C_5}{2}g^{\theta}(t)\leqslant g^{\theta}(t)\left(C_6g^{\eta-\theta}(t)-\frac{C_5}{2}\right), \tag{12}$$

where $C_5=C_1C_3^{\theta(a-1)}C_4^{-a\theta}$ and $C_6=C_2|\Omega|^{1-\eta}R^{s\eta}$. If $u_0(x)$ is sufficiently small so that

$$g(0)=\int_\Omega|x|^{-s}u_0^{l+1}\mathrm{d}x<\left(\frac{C_5}{2C_6}\right)^{\frac{1}{\eta-\theta}}, \tag{13}$$

then by Lemma 1 in Ref. [20], one knows that there are two positive constants σ_1 and σ_2 such that $0\leqslant g(t)\leqslant\sigma_1\mathrm{e}^{-\sigma_2 t}$. Choosing $T_0>\max\left\{0,-\frac{1}{\sigma_2}\ln\left(\frac{1}{\sigma_1}\left(\frac{C_5}{2C_6}\right)^{\frac{1}{\eta-\theta}}\right)\right\}$, one can obtain that, for any $t>T_0$,

$$\frac{\mathrm{d}}{\mathrm{d}t}g(t)+\frac{C_5}{2}g^{\theta}(t)\leqslant g^{\theta}(t)\left[C_6\left(\sigma_1\mathrm{e}^{-\sigma_2 t}\right)^{\eta-\theta}-\frac{C_5}{2}\right]\leqslant 0.$$

That is to say,

$$\frac{\mathrm{d}}{\mathrm{d}t}g^{1-\theta}(t)=(1-\theta)g^{-\theta}(t)\frac{\mathrm{d}}{\mathrm{d}t}g(t)\leqslant-\frac{C_5(1-\theta)}{2},\ t>T_0.$$

Integrating this inequality from T_0 to t yields that $0\leqslant g^{1-\theta}(t)\leqslant g^{1-\theta}(T_0)+\frac{C_5(1-\theta)}{2}(T_0-t)$, which suggests that the solution $u(x,t)$ of problem (1) will vanish as t tends to $T_0+\frac{2g^{1-\theta}(T_0)}{C_5(1-\theta)}$. The proof of Theorem 1 is completed.

Proof of Theorem 2 Inspired by the idea of Ref. [16], we define $f(t)=d^{\frac{1}{m+\alpha-q}}\left(1-\mathrm{e}^{-ct}\right)^{\frac{1}{1-q}}$, $t\geqslant 0$, where $d\in(0,1)$ and $c\in\left(0,\ (m+\alpha-q)d^{\frac{q-1}{m+\alpha-q}}\right)$. Then it is obvious that f satisfies $f(0)=0$, $f(t)\in(0,1)$ and

$$f'(t)+\frac{1}{d}f^{m+\alpha}(t)-f^{q}(t)<0\ \text{ for }\ t>0. \tag{14}$$

Let $v(x,t)=f(t)\psi(x)$, where $\psi(x)$ is the first eigenfunction of the following eigenvalue problem

$$\begin{cases}-\mathrm{div}\left(\psi^{\alpha}|\nabla\psi|^{m-1}\nabla\psi\right)=\lambda_1\psi^{\alpha+1}|\psi|^{m-1}, & x\in\Omega,\\ \psi(x)=0, & x\in\partial\Omega,\end{cases} \tag{15}$$

and is assumed to satisfy $\max\limits_{x\in\bar{\Omega}}\psi(x)=1$. Then by (14) and the first mean value theorem for integrals, for any nonnegative test function $\xi(x,t)\in C_0^{\infty}\left(\bar{\Omega}\times[0,T]\right)$, one has

$$\begin{aligned}I:&=\int_0^T\int_\Omega\left[\left(|x|^{-s}v_\tau-\lambda|\nabla v|^{q}\right)\xi+v^{\alpha}|\nabla v|^{m-1}\nabla v\cdot\nabla\xi\right]\mathrm{d}x\mathrm{d}\tau\\ &=\int_0^T\int_\Omega\left\{\left[|x|^{-s}f_\tau\psi-\lambda f^{q}|\nabla\psi|^{q}\right]\xi+f^{m+\alpha}\psi^{\alpha}|\nabla\psi|^{m-1}\nabla\psi\cdot\nabla\xi\right\}\mathrm{d}x\mathrm{d}\tau\\ &<\int_0^T\int_\Omega\left[|x|^{-s}\left(f^{q}-\frac{1}{d}f^{m+\alpha}\right)\psi+\lambda_1f^{m+\alpha}\psi^{m+\alpha}\right]\xi\mathrm{d}x\mathrm{d}\tau\\ &=T|\Omega|\xi(x^*,\tau^*)\left[|x^*|^{-s}\left(f^{q}(\tau^*)-\frac{1}{d}f^{m+\alpha}(\tau^*)\right)\psi(x^*)+\lambda_1f^{m+\alpha}(\tau^*)\psi^{m+\alpha}(x^*)\right],\end{aligned}$$

where $(x^*,\tau^*)\in\Omega\times(0,T)$. Choosing $d\in\left(0,\ \min\left(1,\frac{\psi^{1-m-\alpha}(x^*)}{2\lambda_1|x^*|^s}\right)\right)$, then one has $I<0$ provided that the first eigenvalue λ_1 of the eigenvalue problem (15) satisfies $\lambda_1\geqslant|x^*|^{-s}\psi^{1-m-\alpha}(x^*)f^{q-m-\alpha}(\tau^*)$. In other words, $v(x,t)$ is a non-extinction weak sub-solution of problem (1). On the other hand, one can prove that $\delta(x,t)=\max\left\{1,\ \|u_0\|_{L^\infty(\Omega)}\right\}$ is a non-extinction super-solution of problem (1). Then by an iterated process, one can claim that problem (1) at least admits a non-extinction weak solution $u(x,t)$ satisfying $v(x,t)\leqslant u(x,t)\leqslant\delta(x,t)$. The proof of Theorem 2 is completed.

Proof of Theorem 3 If $0<m+\alpha=q<1$, then by the arguments in the proof of Theorem 1, one can arrive at (11) with $\theta=\eta=\frac{l+q}{l+1}\in(0,1)$. Namely,

$$\frac{\mathrm{d}}{\mathrm{d}t}g(t)+(C_5-C_6)g^\theta(t)\leqslant 0. \tag{16}$$

If λ is sufficiently small such that

$$C_5-C_6=(l+1)\left(\frac{1}{q+l}\right)^q\left\{lC_3^{\theta(a-1)}C_4^{-a\theta}\left(\frac{1}{q+l}\right)^{m+1-q}-\lambda\left[\varepsilon C_3^{\theta(a-1)}C_4^{-a\theta}+C(\varepsilon)|\Omega|^{1-\theta}R^{s\theta}\right]\right\}$$

is positive, then (16) yields the extinction result of the solution for the case $q=m+\alpha$.

On the other hand, let $v(x,t)=\rho(t)\psi(x)$, where $\psi(x)$ is defined in the proof of Theorem 2 and $\rho(t)=\left[(1-q)t\right]^{\frac{1}{1-q}}$. Then by repeating a similar argument to that in the proof of Theorem 2, one can prove that $v(x,t)$ is a non-extinction sub-solution of problem (1) for some appropriate large λ. Meanwhile, one can also prove that $\delta(x,t)=\max\left\{1,\|u_0\|_{L^\infty(\Omega)}\right\}$ is a non-extinction super-solution of problem (1). Therefore, the standard iterated process can be used to guarantee that problem (1) at least admits a non-extinction weak solution $u(x,t)$ satisfying $v(x,t)\leqslant u(x,t)\leqslant\delta(x,t)$. The proof of Theorem 3 is completed.

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具非线性梯度源和奇异势的快扩散方程解的熄灭

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摘要：本文研究了含非线性梯度源与奇异势项的快扩散方程解的熄灭行为。利用能量估计方法、Hardy-Littlewood-Sobolev不等式和微分不等式技巧，给出了一类具非线性梯度源和奇异势的快扩散方程解在有限时刻发生熄灭现象的充分条件。同时，利用上、下解方法，证明了非熄灭解的存在性。

关键词：熄灭；快扩散方程；梯度源；奇异势

□